SHENTON COLLEGE

Year 11 Mathematics Methods
AEMAM Term 1 2021

Test 1 Counting and Probability

Calculator Free *Formula Sheet Allowed*

Student Name: Solutions

Teacher (circle): Alfonsi Feutrill Loh McRae

Time Allowed: 30 minutes

Calculator Free:	/31

*Attempt **all** questions.*
*All necessary working and reasoning must be shown for **full marks**.*
Marks may not be awarded for untidy or poorly arranged work.

Question 1. **[2, 2, 1, 3, 1 = 9 marks]**

(a) Simplify (i) $0!(6! - 4!)$

$= 1 \times (720 - 24)$ ✓ factorial values

$= 696$ ✓ simplifies

(ii) ${}^{12}C_8$ ✓ correct

$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}$ ✓ correct use of ${}^{n}C_r$

$= 11 \times 9 \times 5$

$= 495$ ✓ simplifies

(b) The sum of all numbers in a certain row of Pascal's triangle is 32.

(i) Write down all the numbers in this row of Pascal's triangle.

1 5 10 10 5 1 ✓ correct row values

(ii) Expand and simplify $(2 - x)^5$

$= 32 + 5(16)(-x) + 10(8)(x^2) + 10(4)(-x^3) + 5(2)(x^4) - x^5$

$= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$ ✓ coefficients ✓ alternating signs ✓ simplifies

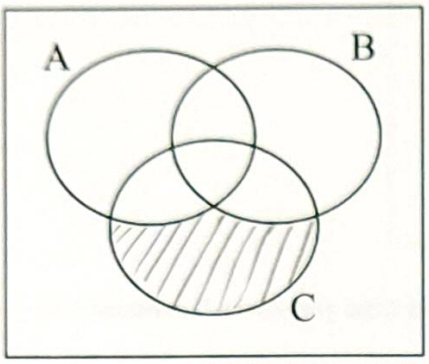
(iii) Hence determine the power of x of the term in $3(2 - x)^5$ that has a coefficient of 30.

4^{th} ✓ correct power

Question 2. **[1, 2, 2 = 5 marks]**

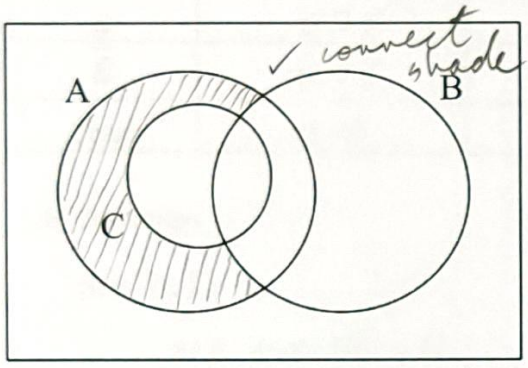
(a) Shade the regions on the Venn diagrams below to represent

(i) $\bar{A} \cap \bar{B} \cap C$



✓ correct shading

(ii) $(A \cup C) \cap (\overline{C \cup B})$



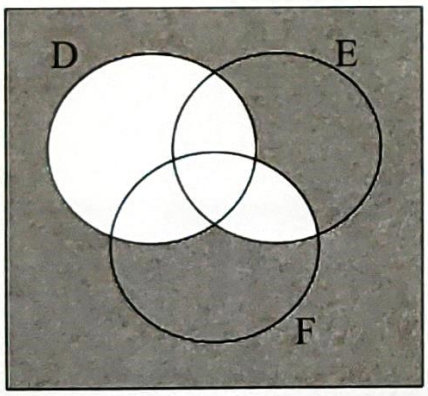
✓ correct shade

$A \cup C = A$

$\therefore A \cap (\overline{C \cup B})$

✓ not needed but mark if shown

(b) Use set notation to describe the shaded region in the Venn diagram below.



$\overline{D \cup (E \cap F)}$

OR

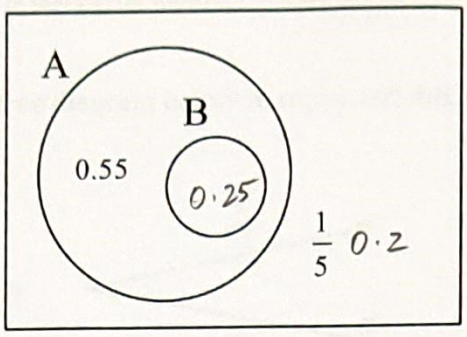
$\bar{D} \cap \bar{E} \cup \bar{F}$

✓ complement

✓ correct use of $\cup$ and $\cap$

Question 3. **[2, 2, 2, 1, 1 = 8 marks]**

Consider the probabilities presented in the Venn diagram below.



(a) Complete the two-way table below using the information in the Venn diagram above.

	A	$\bar{A}$	Total
B	0.25	0	0.25
$\bar{B}$	0.55	0.2	0.75
Total	0.8	0.2	1

✓ ✓

(b) Determine

(i) $P(\bar{A} \cup \bar{B})$

$0.2 + 0.55 + 0.2$ ✓ correct values from table

$= 0.75$ ✓ simplified

(ii) $P(B|A)$

$= \frac{P(B \cap A)}{P(A)} = \frac{0.25}{0.8} = \frac{25}{80}$ or $\frac{5}{16}$

✓ correct use of condition
✓ simplified

(iii) $P((A \cap B)|\bar{B})$ $= \frac{0}{0.25}$

$= 0$ ✓ correct value

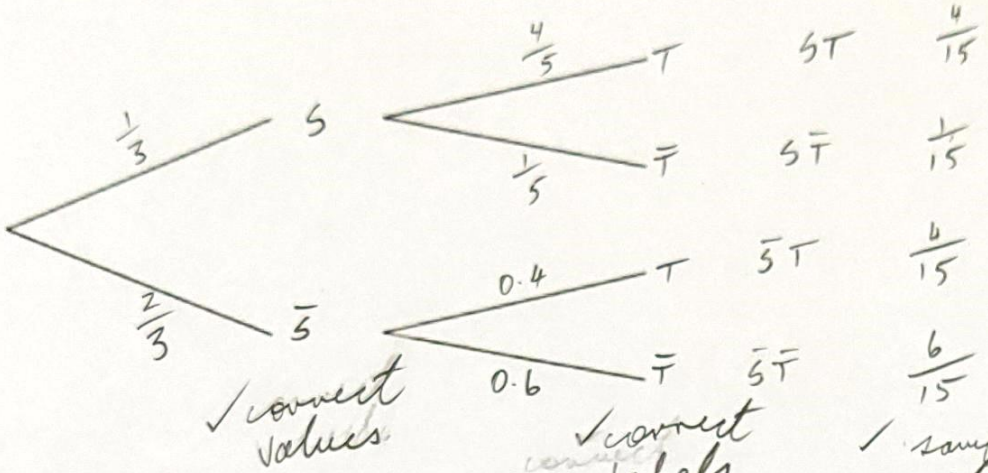
(c) Justify your result in part (b) (iii).

The intersection of A and B is not in the complement of B, hence the numerator in the condition formula is zero. ✓ correct explanation

Question 4. **[3, 1, 1, 1, 3 = 9 marks]**

The probability that it will be sunny (S) tomorrow is $\frac{1}{3}$. If it is sunny tomorrow, then the probability that Jenny will play tennis (T) is $\frac{4}{5}$. If it is not sunny tomorrow, then the probability that Jenny will play tennis drops to 0.4.

(a) Complete the weighted tree diagram below to represent this situation, showing the sample space.



✓ correct values ✓ correct labels ✓ sample space with probabilities

(b) Find the probability that, tomorrow,

(i) It is not sunny, and Jenny does not play tennis.

$\frac{6}{15}$ ✓

(ii) Jenny does not play tennis.

$\frac{7}{15}$ ✓

(iii) Jenny plays tennis.

$\frac{8}{15}$ ✓

Jenny decides to play tennis the following day. She wins the service toss and gets to choose to serve first. She has to select two balls to serve with. There are 5 balls to choose from; 2 Wilson and 3 Dunlop.

(c) Determine the probability that the two balls chosen are of the same brand.

$$\frac{{}^2C_2 \times {}^3C_0 + {}^2C_0 \times {}^3C_2}{{}^5C_2} = \frac{1+3}{10} = \frac{4}{10}$$

✓ correct numerator
✓ correct denominator
✓ simplified

End of Calculator-Free Section

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AEMAM Term 1 2021

Test 1 Counting and Probability

Calculator Assumed *Formula Sheet Allowed*

Student Name: Solutions

Teacher (circle): Alfonsi Feutrill Loh McRae

Time Allowed: 15 minutes

Calculator Assumed: / ~~100~~ 19

*Attempt **all** questions.*
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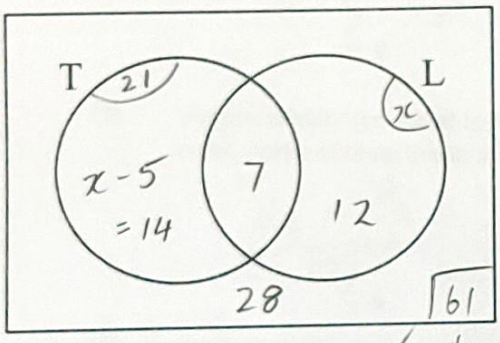
Question 5. **[3, 1, 2 = 6 marks]**

A large kid's party offered hamburgers to their guests, where they had the option of getting tomato and lettuce on their burger. Of the 61 people who attended the party, there were 5 less people who had only tomato on their hamburger as those who had lettuce. There were also 28 people who had neither lettuce nor tomato on their hamburger, and 21 people who had tomato.

(a) Complete the Venn diagram below.



$x - 5 + x = 33$

$2x = 38$

$x = 19$

✓ solves for lettuce

✓ completes Venn diagram

✓ subs. in known information

(b) Determine the probability that a guest had only one of the toppings on their hamburger.

$\frac{26}{61}$ ✓

(c) Determine the probability that a guest only had lettuce on their hamburger, given that they did not have tomato.

$\frac{12}{40}$

✓ correct numerator

✓ correct denominator

Question 6.

[2, 3 = 5 marks]

A bag contains five red, three blue and four yellow balls. Two balls are selected at random from the bag without replacement. Determine the probability (rounded to 4 decimal places) that

(a) the balls chosen are blue and yellow, in any order.

$$\frac{{}^{3}C_1 \times {}^{4}C_1}{{}^{12}C_2} = 0.1818$$

✓ correct numerator
✓ correct probability to 4d.p.

(b) the balls chosen are not the same colour, given that it is known that neither ball is red.

$$P(\overline{\text{same}} \mid \text{neither red}) = \frac{\frac{2}{11}}{\frac{{}^{7}C_2}{{}^{12}C_2}} = \frac{\frac{2}{11}}{\frac{21}{66}} = \frac{12}{21} = 0.5714$$

✓ correct numerator
✓ correct denominator
✓ correct probability to 4d.p

Question 7. **[1, 2, 2, 3 = 8 marks]**

15 laptop computers are stored in a room, of which 11 are in working order but the rest have a fault. An IT technician enters the room an selects some laptops to inspect.

(a) Determine:

(i) the number of ways that four laptops could be chosen from the 15.

$${}^{15}C_4 = 1365$$

✓ correct

(ii) the probability (rounded to 4 d.p.) that when four laptops are chosen at random from the room, three of them are in working order and one has a fault.

$$\frac{{}^{11}C_3 \times {}^{4}C_1}{{}^{15}C_4} = 0.4835$$

✓ correct numerator
✓ correct to 4d.p.

(iii) the probability (rounded to 4 d.p.) that at least one of the four laptops chosen has a fault.

$$1 - \frac{{}^{11}C_4}{{}^{15}C_4} = 0.7582$$

✓ complement process
✓ correct to 4d.p.

When a single laptop is chosen at random from the room, if it is in working order there is a 2% chance it develops a fault upon inspection. If it already has a fault, then there is a 96% chance the technician can fix it.

(b) Determine the probability (rounded to 4 d.p.) that ~~when~~ after inspection, the laptop is in working order.

$$0.98 \times \frac{11}{15} + 0.96 \times \frac{4}{15} = 0.9747$$

✓ complementary of 0.02

✓ multiplication principle
✓ adds both options together and solves.

End of Calculator Assumed Section